HW4

1 Independence of linear and quadratic terms

In HW3 Q3, we consider a hypothesis concerning a contrast of group means in a one-way ANOVA:

$$H_0 : c_1\mu_1 + c_2\mu_2 + \cdots + c_m\mu_m = 0$$

where $c_1 + c_2 + \cdots + c_m = 0$. Define the sample value of the contrast as

$$C \equiv c_1\bar{Y}_1 + c_2\bar{Y}_2 + \cdots + c_m\bar{Y}_m$$

and let

$$C'^2 \equiv \frac{C^2}{\frac{c_1^2}{n_1} + \frac{c_2^2}{n_2} + \cdots + \frac{c_m^2}{n_m}}$$

C'^2 is the sum of squares for the contrast.

We showed

$$\frac{C}{\sqrt{\sigma^2(\frac{c_1^2}{n_1} + \frac{c_2^2}{n_2} + \cdots + \frac{c_m^2}{n_m})}} \sim \mathcal{N}(0,1)$$

And (in class)

$$\frac{SSE}{\sigma^2} \sim \chi^2_{n-m}$$

such that $\frac{(n-m)S_E^2}{\sigma^2} = \frac{SSE}{\sigma^2} \sim \chi^2_{n-m}$. Now please show that $\frac{C}{\sqrt{\sigma^2(\frac{c_1^2}{n_1}+\frac{c_2^2}{n_2}+\cdots+\frac{c_m^2}{n_m})}}$ and S_E^2/σ^2 are independent.

2 Multivariate normal distribution

Let $\mathbf{X} = (X_1, X_2, X_3)^T$ be a trivariate normal random variable with mean $\boldsymbol{\mu}_{\mathbf{X}} = (1, 2, 3)^T$ and covariance matrix

$$\mathbf{\Sigma} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

1. Find the mean and variance of the random variable $Y = 2X_1 - X_2$.
2. Find the mean and covariance matrix of the random vector $(W_1, W_2) = (X_1, X_2 + X_3)$. Are W_1 and W_2 independent?